Using Thermal Inertia of Buildings with Phase Change Material for Demand Response

Zahra Rahimpour\textsuperscript{a}, Gregor Verbič\textsuperscript{a}, Archie C. Chapman\textsuperscript{a}

\textsuperscript{a}School of Electrical and Information Engineering, The University of Sydney, Sydney, Australia
Email: zahra.rahimpour@sydney.edu.au

Abstract

The increasing penetration of distributed generation, such as rooftop PV, is being followed by an uptake of other distributed energy resources such as battery storage and flexible loads, resulting in greater flexibility on the demand side of power systems. Of these resources, space heating and cooling is receiving much attention. At the same time, demand response (DR) programs have demonstrated the ability to manage peak demand and provide energy services at low cost. In particular, combined solar photovoltaic (PV) and battery energy storage (BES) systems in buildings can contribute to reductions or shifts in energy consumption. However, installations of PV-BES systems can have a long payback period. A potential alternative to BES, is to use the thermal inertia of buildings, which acts as an energy storage system. It absorbs heat when the temperature increases and dissipates heat when temperature decreases. However, in Australia, a large number of buildings are lightweight with low thermal inertia. The new promising technology that responds to this problem is using phase change material (PCM). The energy exchange during the phase changing (latent heat) of PCM can be exploited as a form of a thermal resource to improve the thermal inertia of the existing buildings. Achieving the comfort range of indoor temperature in buildings with PCM at minimum electricity cost while maximising self-use of rooftop PV generation that operates the HVAC system can be cast as an optimisation problem. This optimisation problem can solve with home energy management system (HEMS). What makes the problem of the PCM-integrated buildings distinct from other HEMS formulations, is the nonlinear behaviour of the PCM which results in a nonconvex model. The state of art method to handle this nonlinearity is dynamic programming (DP). To this end, this paper proposes: (i) a simple model of a PCM-building’s thermal dynamics that simulates the thermal behaviour of a simple one-zone cubic PCM-building with a rooftop PV panel that produced electricity for operation of the HVAC system and (ii) a solution technique based on DP to solve the corresponding optimisation problem. The simulation results over a summer week in Sydney, demonstrate that PCM has a potential to cut the operation hours of the HVAC system from 9 to 4 hours without violating the indoor comfort range. This reduction includes eliminating shoulder or peak periods as well.

1. Introduction

High penetration of renewable energy resources and advancements in infrastructure technologies have fostered more focus on demand response (DR). DR programs aim to harness the flexibility of electric loads for peak demand shaving, shifting peak demand and other network support services. One of the electric loads that has gained more attention in recent years is heating, ventilation and air conditioning (HVAC). Approximately 50% of the energy used in buildings is for operating HVAC systems, and buildings are responsible for approximately 40% of primary energy consumption and one-third of greenhouse gas emissions globally [1]. Like other parts of the world, Australia follows the same pattern. This has driven the uptake of solar photovoltaic (PV) systems in order to reduce energy expenditure. For example, in Australia over 20% of residential buildings already have a PV system [2]. However, self-consumption is typically low (in the order of 30%), for which battery energy storage (BES) can be a solution. However, BES is still expensive (payback over 10 years), and has a limited lifetime [3].

A potential alternative to BES, is to use the thermal inertia of the buildings. This inherent property of the building’s envelope (walls, roof, floor and fenestration) is defined as the envelope’s ability to store or release excess heating to smooth inside temperature fluctuations. This ability can be considerable, but it relies on the building design and
### Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_{\text{out}}$</td>
<td>Outdoor temperature</td>
</tr>
<tr>
<td>$T_{\text{in}}$</td>
<td>Indoor temperature</td>
</tr>
<tr>
<td>$R_{\text{dw}}$</td>
<td>Total thermal resistance of windows and door</td>
</tr>
<tr>
<td>$R_{\text{out}}$</td>
<td>External thermal resistance of the element</td>
</tr>
<tr>
<td>$R_{\text{in}}$</td>
<td>Internal thermal resistance of the element</td>
</tr>
<tr>
<td>$C_e$</td>
<td>Total thermal capacity of the element</td>
</tr>
<tr>
<td>$Q_{\text{inf}}$</td>
<td>Infiltration heat loss</td>
</tr>
<tr>
<td>$Q_h$</td>
<td>Heater power</td>
</tr>
<tr>
<td>$Q_c$</td>
<td>Cooling power</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of layers in the element</td>
</tr>
<tr>
<td>$R_e$</td>
<td>Total thermal resistance of the element</td>
</tr>
<tr>
<td>$C_e$</td>
<td>Total thermal capacity of the element</td>
</tr>
<tr>
<td>$A_e$</td>
<td>Total element area</td>
</tr>
<tr>
<td>$r_{\text{si}}$</td>
<td>Inside surface thermal resistance of the element</td>
</tr>
<tr>
<td>$r_{\text{so}}$</td>
<td>Outside surface thermal resistance of the element</td>
</tr>
<tr>
<td>$d_i$</td>
<td>Thickness of the $i$th section in the element</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>Density of the $i$th section in the element</td>
</tr>
<tr>
<td>$\lambda_i$</td>
<td>Thermal conductivity of the $i$th section in the element</td>
</tr>
<tr>
<td>$c$</td>
<td>Specific heat capacity of the $i$th section in the element</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Correction factor of thermal resistance</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Surface temperature of the element</td>
</tr>
<tr>
<td>$m_a$</td>
<td>Inside air mass of the building</td>
</tr>
<tr>
<td>$c_a$</td>
<td>Specific heat capacity of inside air of the building</td>
</tr>
<tr>
<td>$c_{\text{pcm}}$</td>
<td>Specific heat capacity of phase change material</td>
</tr>
<tr>
<td>$C_{\text{PCM}}$</td>
<td>Total heat capacity of phase change material</td>
</tr>
<tr>
<td>$T_p$</td>
<td>Melting point of phase change material</td>
</tr>
<tr>
<td>$\pi$</td>
<td>Policy</td>
</tr>
<tr>
<td>$C_k$</td>
<td>Cost or reward at time-step $k$</td>
</tr>
<tr>
<td>$K$</td>
<td>Total number of time-steps</td>
</tr>
<tr>
<td>$k$</td>
<td>Time-step</td>
</tr>
<tr>
<td>$C^\pi_k$</td>
<td>Cost/reward for following $\pi$ from $k$</td>
</tr>
<tr>
<td>$s_k$</td>
<td>State at time-step $k$</td>
</tr>
<tr>
<td>$x_k$</td>
<td>Decision at time-step $k$</td>
</tr>
<tr>
<td>$\omega_k$</td>
<td>Variation of stochastic variable at time-step $k$</td>
</tr>
<tr>
<td>$u,w$</td>
<td>Weighting factor for two parts of the cost function</td>
</tr>
<tr>
<td>$c_g$</td>
<td>Cooling cost</td>
</tr>
<tr>
<td>$P_k$</td>
<td>Varing cooler power</td>
</tr>
<tr>
<td>$p_{\text{PV}}$</td>
<td>Varing PV output</td>
</tr>
<tr>
<td>$T_s$</td>
<td>Desired room temperature</td>
</tr>
<tr>
<td>$V^\pi_k$</td>
<td>Expected future cost/reward for following $\pi$ from $k$</td>
</tr>
<tr>
<td>$ACH$</td>
<td>Air changes per hour</td>
</tr>
</tbody>
</table>

Envelope’s composition. However, in Australia, a large number of buildings are lightweight buildings that have low thermal inertia. As such, cooling and heating of buildings in Australia is challenging for both energy providers and consumers due to the high financial and environmental cost of electricity. Changing the building’s design to use high thermal inertia material in the construction is not a convincing solution due to the high carbon-footprint of brick and large stock of the lightweight buildings. In contrast, phase change material (PCM) is a new building material technology that can be used to store thermal energy and improve the thermal inertia of the buildings. In buildings with PCM as the name implies, the energy exchanged during the phase-change (latent heat) can be exploited as a form of a thermal resource, to cool or heat the building. A common type of this material is paraffin-based, which is available in mat or encapsulated block to easily integrate into the building’s envelope. This material has a significant amount of latent heat that is almost 40 times that of brick with the same mass.\(^1\)

PCM is used to implement DR in buildings as follows. On a summer’s day, a HVAC system can operate during times with excess rooftop PV generation (or shoulder and off-peak price periods for those without PV), a HVAC

---

\(^1\)http://phasechange.com.au
system operates to precool the building to a desired set point temperature, at or below the freezing point of the PCM. When there is less rooftop solar generation, or when the building’s load is larger, or during peak pricing periods, the HVAC system is turned off and the indoor temperature of the building rises to a point where the PCM begins to melt. While the PCM melts, it absorbs heat from building’s interior, thereby maintaining a near-constant indoor temperature. In particular, in order to best exploit the energy storage potential of PCM, the aforementioned precooling (or preheating on a winter’s day) by HVAC system can be scheduled in a way to minimise the electricity cost while maintain the indoor temperature of the building within the comfort range of 22 °C to 26 °C [4].

Like other behind-the-meter distributed energy resources (DERs), such as residential batteries, plug-in electric vehicles, and smart appliances, achieving comfort temperature at minimum cost in a PCM-integrated house is not possible without a home energy management system (HEMS). HEMS have been developed as a means to schedule and coordinate DERs to minimise energy cost while maintaining a suitable level of comfort for end-users [5, 6]. In more detail, HEMS typically comprise: measuring device such as electricity meters; sensors such as temperature sensors; enabling ICT that integrates all devices in HEMS by using advance communication technologies; controllable devices such as HVAC systems and; a management system or software platform and embedded intelligence to operate the HEMS [7].

Within this context, the main focus of this work is a management system or software platform of the HEMS. More specifically, the main contribution of this paper is to schedule a controllable device like HVAC system in buildings with PCM and rooftop PV panel. The objective is to minimise the electricity cost and improve self-use of the PV panel while keeping the indoor temperature in the comfort range. In contrast to current literature [5, 6, 8] in HEMS, this work considers HEMS that has PCM as an alternative to BES. Moreover, like [9–17], this paper demonstrates the effectiveness of PCM on energy performance of the buildings. In other words, it seeks to bridge the gap in the current literature and proposed energy management system in PCM-buildings with more focus on impact of PCM on the buildings’ thermal performance.

Solving the optimization problem, different methods are applied in the literature [5, 6, 18, 19]. Methods such as linear programming (LP) and mixed integer linear programming (MILP) are widely used to solve HEMS problems mainly because of simplicity associated with off-the-shelf solvers, such as CPLEX, Gurobi and MOSEK. The main drawback of these methods is that they optimise linear objective function subject to linear constraints. This is against the desired feature of an optimisation method to deal with nonlinear characteristics of PCM. To consider the nonlinearity of the underlying problem, it is required to transform from MILP to MINLP method. The MINLP formulation captures the nonlinearity of the storage system through curve-fitting a nonconvex and nonlinear function to the data points of the storage system. But this makes the problem complex while the existence of the solution is not guaranteed. Other methods that are extensively used in literature to solve HEMS problem are heuristic methods like particle swarm optimisation (PSO) and genetic algorithm (GA) [18]. These algorithms search semi-randomly within a large population research space to until converge near a solution. In these types of methods there is a risk that the solution ends up in a local optimum instead of the global optimum which means the quality of solution is uncertain.

The state of art method to handle the nonlinearity of PCM is dynamic programming (DP). Moreover, DP is the method to deal with problems that have a sequential structure. In the DP formulation, the problem is modeled as a Markov decision process (MDP), and solved by computing the expected future cost of following an optimal policy (in this problem specific on/off combination of HVAC system) using backward induction. This algorithm is called value iteration of the system state variables (such as indoor temperature) and then an optimal policy can be extracted by selecting the state with the minimum value function using the Bellman optimality condition² [20].

However, in DP, execution time grows exponentially with the increase of a time horizon and the number of variables. Which is called a curse of dimensionality [20]. Since in this paper DP is applied for time horizon of a week, addressing methods to overcome the curse of dimensionality is not included in this work. An interested reader can refer to [5] for more details. This paper shows potential of PCM to reduce operating hours of HVAC system in a typical lightweight building in Australia. Moreover, it introduces HEMS that includes PCM as a storage system. And finally, it present DP as a method to solve the underlying optimisation problem.

The paper progresses as follows: The next section introduces a thermal model of a building, describes how phase change material is incorporated into the model and finally validates the model by benchmarking it against an identical

---

² “An optimal policy has the property that whatever the initial state and initial decision are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first”. 3
Section 3, gives an overview of the HEMS problem in a PCM-building and the solution techniques to solve it. Then, in Section 4, it followed by a case study on a summer week in Sydney. Section 5, concludes and outlines future work.

2. Thermal model of PCM-buildings

The first step towards energy management in PCM-building is deriving a thermal model of a residential building. To capture thermal behaviour of a building, thermal model of PCM-building is simulated and provide the optimisation problem with solid platform to build on. In this section, the type and all the details of the simulated building, along with the computational approach that is used to model the building’s thermal performance are presented. Then it is followed by a brief description of PCM and how it is included in the thermal model of the building. In the last part of this section, the model of the building is validated by benchmarking the model against the identical model in EnergyPlus software. Note that in recent years, several software tools have been developed for simulation studies of buildings’ thermal behaviour. However only a few of these tools have been validated by passing the ANSI-ASHRAE Standard 140 [21]. This standard specifies a method for evaluating a computer program that simulates the thermal energy in buildings. EnergyPlus is a widely known software that complies with the standard.

2.1. Thermal model of lightweight building

The applied modelling method in this work is the RC lumped model approach [22–24]. Therefore, each element of the building is simulated as an RC electric circuit. In this work, for simplicity, all elements of the roof, walls and floor are lumped together as a united 2R1C model (two lumped resistances and one lumped capacitance) as shown in Fig. 1. In more detail, the parallel \( R_{\text{dw}} \) representing fenestration of the building (such as windows and doors) and is the sum of the thermal resistances of door and windows. The infiltration heat loss, heating power and cooling power that enters to the building are represented by \( Q_{\text{inf}}, Q_h \) and \( Q_c \) respectively. To simulate the indoor air, \( m_a c_a \) is considered in the model while the \( C_e \) is thermal inertia (capacity) of envelope that we aim to improve and use as storage system. In this model, total thermal resistance of the building’s element is divided into two resistance of \( R_{\text{in}} \) and \( R_{\text{out}} \) that are named inner and outer resistances respectively. And finally, \( T_{\text{in}} \) and \( T_{\text{out}} \) are representing indoor and outdoor temperature respectively.

The studied building is a simple, one-zone cube with 8m \( \times \) 6m \( \times \) 2.7m dimension and a total floor area of 48 m\(^2\). The building is equipped with HVAC system with rating power of 7 kW that draws its electricity from PV panel on the roof. The PV panel maximum output is about 10 kW. In an effort to reflect real lightweight building in Australia, details of a common lightweight building in Australia that are presented in [15] are adopted in this work [4, 15]. Therefore, the building (from outside to inside) is made up of three layers of rendered fibro-cement, a timber stud wall containing insulation batts, and plaster board [15]. The properties of the building’s material are shown in Table. 1.

Computing inner (\( R_{\text{in}} \)) and outer resistances (\( R_{\text{out}} \)) of Fig. 1, necessitates calculation of total thermal resistance (\( R_e \)) and total thermal capacitance (\( C_e \)) of the buildings’ element based on the given (1) and (2):

\[
R_e = \left( r_{\text{di}} + r_{\text{so}} + \sum_{i=1}^{n} d_i / h_i \right) / A_e
\]  

(1)
Table 1: Building elements composition and its material properties [4, 15].

<table>
<thead>
<tr>
<th>Element</th>
<th>d (m)</th>
<th>λ (W/mK)</th>
<th>ρ (kg/m³)</th>
<th>c (J/kg K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rendered fibro-cement</td>
<td>0.005</td>
<td>0.25</td>
<td>1150</td>
<td>840</td>
</tr>
<tr>
<td>Timber studwall containing insulation</td>
<td>0.09</td>
<td>0.15</td>
<td>650</td>
<td>1200</td>
</tr>
<tr>
<td>Plaster board</td>
<td>0.01</td>
<td>0.25</td>
<td>950</td>
<td>840</td>
</tr>
</tbody>
</table>

\[
C_e = A_e \sum_{j=1}^{n} (d_j \rho_j c_j) \quad (2)
\]

In [4] which is an international reference book for buildings, \( r_{si} \) and \( r_{so} \) have values of 0.13 m² K/W and 0.04 m² K/W, the same values are used here. Refer to (3) and (4), to obtain the values of \( R_{in} \) and \( R_{out} \), total resistance of the element is multiplied by a factor \( (\alpha) \) [22]. This factor varies among the available literature and this cause ambiguity for the researchers in this area. In [25], the author performed an extensive investigation by frequency response analysis of the building and conducting experiments on a set of houses and office rooms. As an outcome, he estimated factor for different elements of building with various boundary conditions. Based on his results, \( \alpha=0.01 \) is used in this model.

\[
R_{in} = \alpha R_e \quad (3)
\]

\[
R_{out} = (1 - \alpha) R_e \quad (4)
\]

Finally, the thermal time-derivation equations (dot above variables shows time-derivation of that variable) of the model are given in (5), (6) and (7):

\[
\dot{T}_e = \frac{1}{C_e + C_{PCM}} \left( \frac{T_{in} - T_e}{R_{in}} + \frac{T_{out} - T_e}{R_{out}} \right) \quad (5)
\]

\[
\dot{T}_{in} = \frac{1}{m_a c_a} \left( \frac{T_{out} - T_{in}}{R_{dpw}} + \frac{T_e - T_{in}}{R_{in}} + \dot{Q}_c + \dot{Q}_{inf} \right) \quad (6)
\]

where:

\[
\dot{Q}_{inf} = \frac{m_a c_a (T_{out} - T_{in}) ACH}{3600} \quad (7)
\]

Since in this work, only a cooling system is studied, in (6), \( \dot{Q}_h \) is equal to zero. Moreover, based on [4], the assumed value for \( ACH \) is considered as 0.3 /h. Note that (5), (6) and (7) can not be solve by fixed-step solvers since as shown in [26], the outputs become unstable. For solving these differential equations in the Matlab, solver ode23s with variable time step is adopted in this work. This solver is based on the Runge-Kutta method which uses a variable, continuously adjusted time step.

In the next section, PCM is added to the current model to achieve a model that we aim to use in the optimisation problem.

2.2. Including PCM in thermal model

Including PCM in the thermal model of building is not a straightforward task because of solid-liquid phase transition. In this subsection, the simple way of modelling PCM is presented. From thermodynamic point of view, the PCM absorbs (melting) or releases energy as heat (freezing) due to enthalpy changes when the temperature varies within a certain range. This heat is called latent heat, the latent heat is the main useful property of phase change material. The high amount of latent heat in PCM makes this material a supreme thermal storage.

For a typical PCM, the specific heat capacity variation by temperature is as shown in Fig. 2 [10]. The temperature range of 22°C to 28.5°C is where the phase changing occurs. At 27.6°C (the melting point), the specific heat capacity

---

Footnote:

1 ACH is abbreviation of Air Changes per Hour which is a measure of changes in air volume for specific space.
The type of PCM that is utilized in this research is honeycomb PCM that is studied in [11]. The reason is the mathematical equations of specific heat capacity of PCM are provided in [11]. Therefore it can be easily plugged into the thermal model equations. The honeycomb PCM consists of a honeycomb matrix that is enclosed in aluminium sheaths. The formulas of specific heat capacity are given in (8a) and (8b).

\[
c_{pcm} = 1200 + 18800e^{-\frac{1.5(T_p - T)}{T}} \quad \text{if} \quad T < T_p
\]

\[
c_{pcm} = 1300 + 18700e^{-4\left(T_p - T\right)^2} \quad \text{if} \quad T \geq T_p
\]

where \(T_p\) is the melting point of the PCM. Due to the discontinuous form of (8a) and (8b), they cannot be directly substituted continuous equations of in (5), (6) and (7).

To overcome this, the equations are curve fitted with a continuous polynomial function and the resulted function is applied as \(c_{pcm}\) or specific heat capacity of PCM in the equations. Specifically, the total heat capacitance of utilized PCM \((C_{PCM})\) in the building can be calculated from the properties of PCM that are shown in Table 2.

To include PCM in the thermal model, the thickness of timber wall is adjusted in a way that the total thermal resistance of building remains unchanged. Then the PCM layer is placed underneath the plaster layer, which is in the vicinity of timber wall. This means that the two layers with high thermal inertia (PCM layer and timber wall) can be lumped together as a single capacitance. Therefore, the thermal model of the building with PCM is based on the electric circuit shown in Fig. 3. The only difference between Fig. 3 and Fig. 1 is that \(C_{PCM}\) is added to thermal capacitance of the building.

All equations (5) to (7) are applicable to the PCM-building, and only (7) is changed. In more detail, \(C_e\) is replaced by \(C_e + C_{PCM}\) to form (9):

\[
T_e = \frac{1}{C_e + C_{PCM}} \left( \frac{T_{in} - T_c}{R_{in}} + \frac{T_{out} - T_c}{R_{out}} \right)
\]

In the next section, the validity of the described model is checked by benchmarking against EnergyPlus software.
2.3. Benchmarking thermal model against the identical model in EnergyPlus

To check the validity of the proposed thermal model, an identical model is built in EnergyPlus software. The simulations are run for a typical summer month (1/02-28/02). Fig. 4 and Fig. 5, presents that in both NOPCM and PCM-included building the outputs of two models are matching well with the results of model in EnergyPlus. Root-mean-square error (RMSE) is used as a measure to show the difference between thermal model in the Matlab versus the model in EnergyPlus. It shows that maximum value of RMSE in both NOPCM and PCM, over the different horizons, is almost less than 0.8 °C which is acceptable regarding the model uncertainties and also human sensitivity.

In the next section, the differential equations of this validated model of building will be used as inputs to the optimisation problem.

3. Home energy management in PCM-buildings

The considered smart home in this paper consists of PCM that is integrated in all elements of the building’s envelope and a PV system that provides electricity for the HVAC system that in installed in the building. In addition, the building is connected to the electricity grid to provide backup for the HVAC system operation. As mention in the Introduction, PCM has a nonlinear characteristic, so trying to solve the corresponding optimisation problem with the existing MINLP or heuristic solvers is computationally too expensive. Whereas DP can properly incorporate nonlinear behaviour of PCM. To be able to solve a problem with DP, it is required to formulate as a Markov decision process.

Figure 3: 2RC lumped model of PCM-building

Figure 4: Indoor temperature of building without PCM benchmarked against EnergyPlus results.
(MDP). A process can be captured as MDP, if it has a Markov property\(^4\). The underlying optimisation problem in PCM-building is a sequential decision making process. This means that the problem can be formulated as a Markov decision process (MDP). The objective of the HEMS with PCM is to minimise energy cost over decision horizon without sacrificing the customers comfortibility. Given this context, in this section, first we formulate the optimisation problem as an MDP, then describe the assumptions are taken in this work and finally use DP as a solution technique to solve the corresponding MDP.

3.1. Optimisation problem

In general, an MDP consists of a state space, \(s \in S\), a decision space, \(x \in X\), transition functions and contribution functions. The variables \(k\) and \(K\) denote a particular time-step and the total number of time-steps respectively, therefore \(k \in \{1 \ldots K\}\). A state variable, \(s_k \in S\), contains the information that is necessary and sufficient to make the decisions and compute costs, rewards and transitions. The decision variable, \(x_k \in X\), is a action for transition from one state to other state over the decision horizon for all time steps. The random variable, \(\omega_k \in \Omega\), depends on either weather or inhabitants’ behavioural patterns \([28]\).

As discussed in the Introduction section, the condition of problem is supposed to be deterministic, therefore in this problem \(\omega_k\) can be ignored. The MDP form of the problem is given by:

\[
\min_{\pi} \mathbb{E} \left\{ \sum_{k=0}^{K} C_k(s_k, x_k, \omega_k) \right\}
\]

s.t.

- thermal comfort
- and thermal energy balance constraints \((10)\)

where \(\pi\) is a policy, a choice of action for each state, \(\pi : S \rightarrow X\), in this work policy is on/off status of the cooling system. Function \(C_k(s_k, x_k, \omega_k)\) is the contribution (i.e cost/reward of energy incurred at a given time-step \(k\) which accumulates over time \([28]\).

In this work, \((11)\) is used as a contribution function that is described in \((10)\) which has a general form of the contribution function. To capture both comfortability and electricity cost, the contribution function includes two

---

\(^4\)A process has a Markov property, if future state depends only on current time step not on the sequence of events that preceded it \([27]\).
weighting factors of $u$ (cooling cost) and $w$ (cost of deviating from a desired set point of cooling system which is $T_s$) to depict the importance of each part of the cost function to the home owner. The value of this factor is between 0 to 1, based on home users preference. The variable $P_k$ is the amount of electricity from the power grid that operates HVAC system, whereas $p_{vk}$ is electricity that is drawn from PV panel for the cooling system’ operation.

$$\sum_{k=0}^{K} C_{rk}^k(s_k, x_k, \omega_k) = \sum_{k=0}^{K} uc_{rk}^k (P_k - p_{vk}) + w \left( |T_{room,k} - T_s| \right)$$

Back to (10), $s_{k+1} = s^M(s_k, x_k)$ describes the evolution of states from time step $k$ to next time step $k + 1$, where $s^M$ is the underlying mathematical model of the studied system. In this problem, the model which is thermal model of the studied building described by (5),(6) and (7). With this in hand, next part gives an overview of the assumptions that are taken to solve the energy management problem in building with PCM.

3.2. Assumptions

These are the summary of assumptions taken in this work:

- As aforementioned, increasing time horizon or state variables increases execution time of the simulation exponentially. At this initial work, for simplicity, only indoor temperature is monitored throughout the simulations and optimisation algorithms.

- In order to simplify further, the input variables to HEMS like weather data is considered deterministic.

- We assume that the exact electricity prices are available before the start of the decision horizon from an residential DR aggregator/retailer. The daily electricity tariff used in the simulation is illustrated in Fig. 6.

- PV generation output is collected during the Smart Grid Smart City project in NSW, Australia [29]. The electricity grid provides backup power for operation of the HVAC system when the electrical power generated from the PV system is less than the required power for operation of the HVAC system.

3.3. Solution techniques

What is given in (11) is only the instantaneous cost that results from the decision that we take at each time step. To cast the optimisation problem of HEMS, it is needed to include the expected future cost from the state we end up at the next time step as well. Therefore, the optimisation problem is formulated in the form of (12). DP solves the optimisation problem of the form in (12) by computing a value function $V^\pi(s_k)$, which is the expected future discounted cost of following a policy, $\pi$, starting in state, $s_k$, and is given by [28]:

$$V^\pi(s_k) = \sum_{s' \in S} P(s'|s_k, x_k, \omega_k) \left[ C(s_k, x_k, s') + V^\pi(s') \right]$$

Figure 6: Electricity tariff over a day.
where \( P(s'|s_k, x_k, \omega_k) \) is the transition probability of landing on state \( s' \) from \( s_k \) if we take action \( x_k \). An optimal policy, \( \pi^* \), is one that optimises (12). It can be found by recursively computing the optimal value function, \( V^\pi_k(s_k) \), by using Bellman’s optimality condition:

\[
V^\pi_k(s_k) = \min_{x_k \in X_k} \left\{ C_k(s_k, x_k(s_k)) + \mathbb{E}\left\{ V^\pi_{k+1}(s')|s_k\right\} \right\}.
\]  

(13)

The expression in (13) is typically computed using backward induction called value iteration (VI) and then an optimal policy is extracted from the value function by selecting a minimum value action for each state. To describe this in a simple way, in VI for \( k \) steps, the desired states in \( k + 1 \) step is set to lower value while the undesired and out of comfort bound states are penalized by allocating higher values. Then, for all possible states at time \( k \), the VI algorithm moves backward in time and in each time step by applying Bellmann optimality principal, the minimum value function is computed for different states of each time step. In a final step of backward induction corresponding to the initial starting point, all value function calculations will converge to a one specific value function at the end. By tracing the calculated minimum value function forward over a given time horizon, the optimal policy is extracted. In the next section, using described DP technique, results of the optimisation problem of the studied building with and without PCM over a summer week are presented and discussed.

4. Implementation

Fig. 7 shows outputs of the optimisation problem that is formulated in Section 3, along with input outdoor temperature and PV output. The demonstration simulates a typical summer week in Sydney. The decision horizon is 168 hours and slot length one hour, giving 168 time-slots. The outdoor temperature is an average temperature that varies between minimum temperature of 19.7 °C and maximum temperature of 34.1 °C. Moreover, PV output has a maximum value of about 10 kW which is around 2pm for almost all days over the summer week. From indoor
temperature plot, it is evident that using PCM can significantly reduce the temperature fluctuations. This means the HVAC system operates for less hours than a case without PCM, to keep the indoor temperature in the comfort range of 22 °C to 26 °C. Moreover, it is easy to observe that indoor temperature of building with PCM for hours of 16-24, 44-57 and 91-102, is constant and this where phase changing occurs. During this phase changing, PCM absorbs heat from the building’s inerita and keeps the indoor temperature in the desired range whereas in building without PCM, during those hours of phase changing, the HVAC system operation is required to maintain the temperature within the comfort range. The last part illustrated in Fig. 7, is optimal (on/off) policy of the cooling system that results in minimum cost to the home owner and at the same time keeps the indoor temperature in the comfort range. In more detail, comparing optimal policy of PCM-building versus building without PCM gives the following observations. On day 1, PCM eliminates one hour of the HVAC system operation during the shoulder period. On day 4, PCM reduces 2 hours of the cooling system that are one hour from the shoulder and one hour from the peak period. On day 5, it cuts one hour of the cooling system operation and finally on day 7, it prevents the cooling system to be used during one hour of the shoulder and one hour of the peak period. In overall, using PCM for a typical summer week, reduces operating hours of the HVAC system from 9 hours to 4 hours while shaves loads during the shoulder and the peak hours as well. Based on the tariff shown in Fig. 6, PCM saves almost 45$ over a week. This saving will be higher, if we consider the benefit that can be obtained from sending the electrical power from the PV system back to the grid. However, this is not included in the scope of this paper.

5. Conclusion

In this paper, we introduce PCM as a potential replacement to the current BES in the HEMS that consists of the HVAC system that mainly relays on rooftop solar PV. We demonstrate how existing technique of DP can be used to solve the underlying optimisation problem. Finally, our preliminary results of simulation illustrate significant impact of PCM in reducing working hours of the HVAC system on a typical summer week in Sydney. As aforementioned, by increasing state variables, time horizon or resolution of the time horizon, the execution time of DP grows exponentially. Future work will mainly focus on developing a computationally efficient HEMS to overcome the highly cited drawback of DP: curse of dimensionality. Moreover, we will incorporate variables such as weather condition, solar profile and inhabitant’s behaviour as stochastic variables in order to improve the quality of the solution.

6. References


[25] G. Masy, Definition and validation of a simplified multi-zone dynamic building model connected to heating system and hvac unit, Phd, Université de Liege.


